

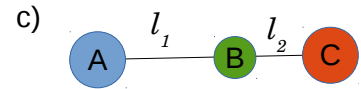
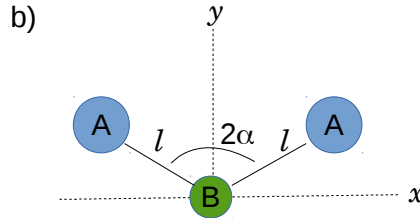
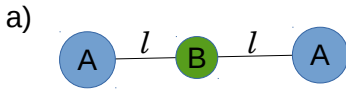
NHT3064-15- Física Ondulatória (2.2017)

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4ª Lista de exercícios

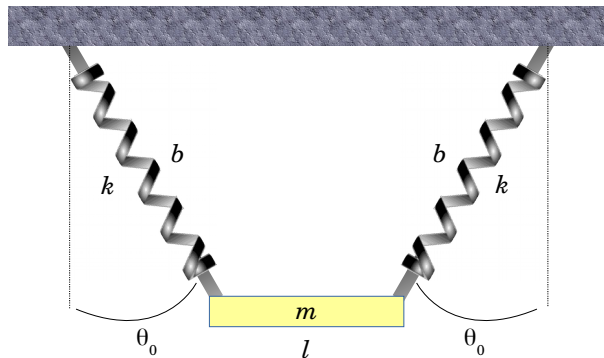
1. Uma partícula se move num campo gravitacional vertical constante ao longo da curva $y=ax^2$, onde y é a direção vertical. Encontre a equação de movimento para pequenas oscilações ao redor da posição de equilíbrio.

2. Obtenha os modos normais de vibração para as seguintes moléculas:



3. Obtenha os modos normais de vibração para as molécula do exercício 2, considerando agora que o átomo B está conectado à origem por uma mola de constante de mola k .

4. Uma barra uniforme de comprimento l e massa m é suspensa por duas molas de comprimento de equilíbrio b e contante de mola k , como indicado na figura. Em repouso, as molas fazem ângulo θ_0 com a vertical. Encontre os modos normais no plano, para pequenas oscilações.



5. Encontre os modos normais de vibração para as cadeias linear monoatômica e diatômica, com espaçamento a . Obtenha os modos acústicos e ópticos, caso existam, no centro e na borda da primeira zona de Brillouin.

6. Um diapasão oscila com uma frequência de 440 Hz. Se a velocidade de propagação do som no ar for de 340 m/s, determine o comprimento de onda e o número de onda do som. Escreva as expressões que representam a onda.

7. Uma mola não esticada tem comprimento de 10 cm e massa de 20 g. Quando se suspende na mola um corpo de massa 200 g, ela alonga-se 0,3 cm. Qual é a velocidade das ondas longitudinais ao longo da mola?

8. Um cabo de aço de 0,2 mm de diametro está sujeito a uma tensão de 200 N. a) Calcule a velocidade de propagação das ondas transversais no cabo. b) Escreva a expressão de uma onda transversal cuja frequência é 400 Hz.

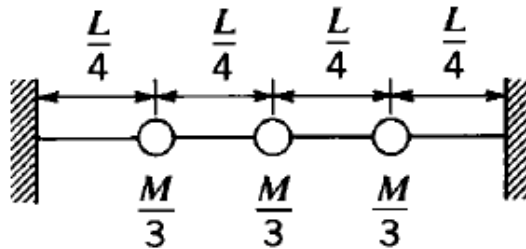
FRENCH:

6-1 A uniform string of length 2.5 m and mass 0.01 kg is placed under a tension 10 N.

(a) What is the frequency of its fundamental mode?

(b) If the string is plucked transversely and is then touched at a point 0.5 m from one end, what frequencies persist?

6-2 A string of length L and total mass M is stretched to a tension T . What are the frequencies of the three lowest normal modes of oscillation of the string for transverse oscillations? Compare these frequencies with the three normal mode frequencies of three masses each of mass $M/3$ spaced at equal intervals on a massless string of tension T and total length L .



6-5 A stretched string of mass m , length L , and tension T is driven by two sources, one at each end. The sources both have the same frequency ν and amplitude A , but are exactly 180° out of phase with respect to one another. What is the smallest possible value of ω consistent with stationary vibrations of the string?

6-6 A uniform rod is clamped at the center, leaving both ends free.

(a) What are the natural frequencies of the rod in longitudinal vibration?

(b) What is the wavelength of the n th mode?

(c) Where are the nodes for the n th mode?

6-9 A room has two opposing walls which are tiled. The remaining walls, floor, and ceiling are lined with sound-absorbent material. The lowest frequency for which the room is acoustically resonant is 50 Hz.

(a) A complex noise occurs in the room which excites *only* the lowest two modes, in such a way that each mode has its maximum amplitude at $t = 0$. Sketch the appearance, for each mode *separately*, of the displacement versus x at $t = 0$, $t = 1/200$ sec, and $t = 1/100$ sec.

(b) It is observed that the maximum displacement of dust particles in the air (which does not necessarily occur at the same time at each position!) at various points between walls is as follows:

| | | | |
|--------------|----------|----------|----------|
| x | $L/4$ | $L/2$ | $3L/4$ |
| ξ_{\max} | $+10\mu$ | $+10\mu$ | -10μ |

What are the amplitudes of each of the two separate modes?

6-14 Find the Fourier series for the following functions ($0 \leq x \leq L$):

(a) $y(x) = Ax(L - x)$.

(b) $y(x) = A \sin(\pi x/L)$.

(c) $y(x) = \begin{cases} A \sin(2\pi x/L), & 0 \leq x \leq L/2 \\ 0, & L/2 \leq x \leq L. \end{cases}$

6-15 Find the Fourier series for the motion of a string of length L if

(a) $y(x, 0) = Ax(L - x)$; $(\partial y/\partial t)_{t=0} = 0$.

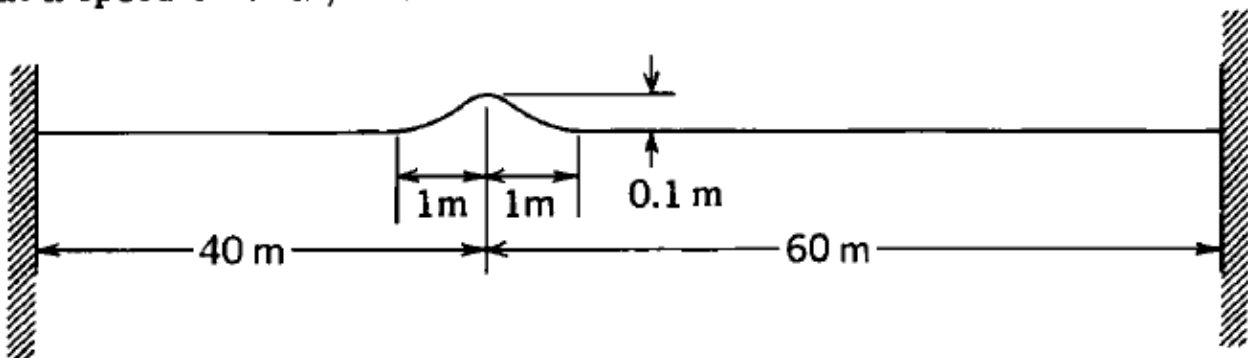
(b) $y(x, 0) = 0$; $(\partial y/\partial t)_{t=0} = Bx(L - x)$.

7-11 Suppose that a traveling wave pulse is described by the equation

$$y(x, t) = \frac{b^3}{b^2 + (x - vt)^2}$$

with $b = 5 \text{ cm}$ and $v = 2.5 \text{ cm/sec}$. Draw the profile of the pulse as it would appear at $t = 0$ and $t = 0.2 \text{ sec}$. By direct subtraction of ordinates of these two curves, obtain an appropriate picture of the transverse velocity as a function of x at $t = 0.1 \text{ sec}$. Compare with what you obtain by calculating $\partial y / \partial t$ at an arbitrary t and then putting $t = 0.1 \text{ sec}$.

7-12 The figure shows a pulse on a string of length 100 m with fixed ends. The pulse is traveling to the right without any change of shape, at a speed of 40 m/sec.



(a) Make a clear sketch showing how the transverse velocity of the string varies with distance along the string at the instant when the pulse is in the position shown.

(b) What is the maximum transverse velocity of the string (approximately)?

(c) If the total mass of the string is 2 kg, what is the tension T in it?

(d) Write an equation for $y(x, t)$ that numerically describes sinusoidal waves of wavelength 5 m and amplitude 0.2 m traveling to the left (i.e., in the negative x direction) on a very long string made of the same material and under the same tension as above.

7-23 One end of a stretched string is moved transversely at constant velocity u_y for a time τ , and is moved back to its starting point with velocity $-u_y$ during the next interval τ . As a result, a triangular pulse is set up on the string and moves along it with speed v . Calculate the kinetic and potential energies associated with the pulse, and show that their sum is equal to the total work done by the transverse force that has to be applied at the end of the string.

7-24 Consider a longitudinal sinusoidal wave $\xi = \xi_0 \cos 2\pi k(x - vt)$ traveling down a rod of mass density ρ , cross-sectional area S , and Young's modulus Y . Show that if the stress in the rod is due solely to the presence of the wave, the kinetic-energy density is $\frac{1}{2} \rho S (\partial \xi / \partial t)^2$, and the potential-energy density is $\frac{1}{2} Y S (\partial \xi / \partial x)^2$. Thus show that the kinetic energy per wavelength and the potential energy per wavelength both equal $\frac{1}{4} (\rho S \lambda) u_0^2$, where u_0 is the maximum particle velocity $(\partial \xi / \partial t)$.