**NHT3064-15- Física Ondulatória (2.2017)**
**Prof. Herculano Martinho**
**3ª Lista de exercícios**

**1.** A equação $m\ddot{x}+kx=F_0 sen\,\omega t$ descreve o movimento de um oscilador harmônico simples não amortecido sob a ação de uma força de frequencia ω.
a) Mostre que a solução para o estado estacionário é dada por

$$x=\frac{F_0 sen(\omega t)}{m(\omega_0^2-\omega^2)} \text{ , onde } \omega_0^2=k/m$$

b) Faça um desenho para ilustrar o comportamento de *x(ω)* e note que a mudança de sinal quando *ω* passa por $\omega_0$ define uma mudança de fase de π rad no deslocamento.
c) Mostre que a solução geral é dada por

$$x=\frac{F_0 sen(\omega t)}{m(\omega_0^2-\omega^2)}+A\cos(\omega_0 t)+B sen(\omega_0 t)$$ , onde *A* e *B* são constantes.

d) Se $x=\ddot{x}=0$ em $t=0$ , mostre que

$$x=\frac{F_0}{m(\omega_0^2-\omega^2)}\left(sen(\omega t)-\frac{\omega}{\omega_0}sen(\omega t)\right)$$

e) Mostre que perto da ressonância,

$$x=\frac{F_0}{2m\omega_0^2}[sen(\omega_0 t)-\omega_0 t\cos(\omega t)]$$

f) Faça um esquema da solução e) indicando as contribuições relativas dos termos envolvidos.

**2.** Encontre a expressão para a frequencia da máxima aceleração de um oscilador mecânico simples amortecido sob a ação de força $F=F_0\cos(\omega t)$ . Mostre que se $b=\sqrt{km}$ , então a amplitude da aceleração em $\omega_0$ é igual ao limite da amplitude da aceleração para altas frequencias.

**3.** Considere o movimento de uma carga elétrica amortecida (frequencia de amortecimento γ) ligada sob a ação de um campo elétrico oscilante $E=E_0\cos(\omega t)$ <u>e em fase com este</u>.
a) Para uma densidade eletrônica *n*, mostre que a permitividade elétrica vale

$$\epsilon=\epsilon_0+\frac{ne^2 m(\omega_0^2-\omega^2)}{m^2(\omega_0^2-\omega^2)+\omega^2\gamma^2}$$

b) Encontre a expressão para a taxa média de absorção de energia por unidade de volume.
c) Considere que este elétron emite luz em comprimento de onda de 600 nm e que o fator de qualidade envolvido na emissão é Q = 5 x $10^7$. Qual é a largura de banda da ressonância?

**4.** Mostre que a largura de banda da curva de absorção de ressonância define o intervalo de ângulo de fases tal que $tg\,\delta=\pm 1$ .

**5.** Demonstre que as equações de Bloch no referencial girante associado ao campos $B_x=B_1\cos\omega t$ e $B_y=B_1 sen\,\omega t$ ficam

$$\frac{dM_x}{dt}=(\omega_0-\omega)M_y-\frac{M_x}{T_2}$$

$$\frac{dM_y}{dt}=-(\omega_0-\omega)M_x+\omega M_z-\frac{M_y}{T_2}$$

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$$\frac{dM_z}{dt} = -\omega_1 M_y - \frac{M_z - M_0}{T_1}$$

onde $\omega_0 = \gamma B_0$ e $\omega_1 = \gamma B_1$ . Encontre as suas soluções.

**6.** Uma amostra contendo um núcleo magnético está em equilíbrio térmico em campo magnético zero. Um campo magnético é ligado ao longo do eixo z. Após um tempo a direção do campo é instantaneamente alterada para o eixo x. Faça uma previsão do movimento das três componentes da magnetização do spin nuclear através desta sequência de eventos, descrevendo a dinâmica dos spins a nível microscópico.

Problemas - French

*3–12* The motion of a linear oscillator may be represented by means of a graph in which $x$ is shown as abscissa and $dx/dt$ as ordinate. The history of the oscillator is then a curve.

(a) Show that for an undamped oscillator this curve is an ellipse.

(b) Show (at least qualitatively) that if a damping term is introduced one gets a curve spiraling into the origin.

*3–14* An object of mass 0.2 kg is hung from a spring whose spring constant is 80 N/m. The object is subject to a resistive force given by $-bv$, where $v$ is its velocity in meters per second.

(a) Set up the differential equation of motion for free oscillations of the system.

(b) If the damped frequency is $\sqrt{3}/2$ of the undamped frequency, what is the value of the constant $b$?

(c) What is the $Q$ of the system, and by what factor is the amplitude of the oscillation reduced after 10 complete cycles?

*3–15* Many oscillatory systems, although the loss or dissipation mechanism is not analogous to viscous damping, show an exponential decrease in their stored *average* energy with time, $\overline{E} = \overline{E}_0 e^{-\gamma t}$. A $Q$ for such oscillators may be defined using the definition $Q = \omega_0/\gamma$, where $\omega_0$ is the natural angular frequency.

(a) When the note "middle C" on the piano is struck, its energy of oscillation decreases to one half its initial value in about 1 sec. The frequency of middle C is 256 Hz. What is the $Q$ of the system?

(b) If the note an octave higher (512 Hz) takes about the same time for its energy to decay, what is its $Q$?

(c) A free, damped harmonic oscillator, consisting of a mass $m = 0.1$ kg moving in a viscous liquid of damping coefficient $b$ ($F_{\text{viscous}} = -bv$), and attached to a spring of spring constant $k = 0.9$ N/m, is observed as it performs damped oscillatory motion.

Its average energy decays to $1/e$ of its initial value in 4 sec. What is the $Q$ of the oscillator? What is the value of $b$?

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*4–4* A block of mass $m$ is connected to a spring, the other end of which is fixed. There is also a viscous damping mechanism. The following observations have been made on this system:

(1) If the block is pushed horizontally with a force equal to $mg$, the static compression of the spring is equal to $h$.

(2) The viscous resistive force is equal to $mg$ if the block moves with a certain known speed $u$.

(a) For this complete system (including both spring and damper) write the differential equation governing horizontal oscillations of the mass in terms of $m$, $g$, $h$, and $u$.

Answer the following for the case that $u = 3\sqrt{gh}$:

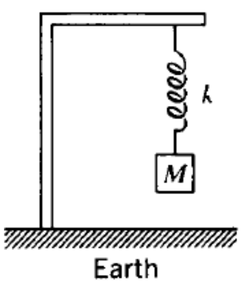
(b) What is the angular frequency of the damped oscillations?

(c) After what time, expressed as a multiple of $\sqrt{h/g}$, is the *energy* down by a factor $1/e$?

(d) What is the $Q$ of this oscillator?

(e) This oscillator, initially in its rest position, is suddenly set into motion at $t = 0$ by a bullet of negligible mass but nonnegligible momentum traveling in the positive $x$ direction. Find the value of the phase angle $\delta$ in the equation $x = Ae^{-\gamma t/2}\cos(\omega t - \delta)$ that describes the subsequent motion, and sketch $x$ versus $t$ for the first few cycles.

(f) If the oscillator is driven with a force $mg \cos \omega t$, where $\omega = \sqrt{2g/h}$, what is the amplitude of the steady-state response?




*4–6* Imagine a simple seismograph consisting of a mass $M$ hung from a spring on a rigid framework attached to the earth, as shown. The spring force and the damping force depend on the displacement and velocity relative to the earth's surface, but the dynamically significant acceleration is the acceleration of $M$ relative to the fixed stars.

(a) Using $y$ to denote the displacement of $M$ relative to the earth and $\eta$ to denote the displacement of the earth's surface itself, show that the equation of motion is

$$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega_0^2 y = -\frac{d^2\eta}{dt^2}$$

(b) Solve for $y$ (steady-state vibration) if $\eta = C\cos\omega t$.

(c) Sketch a graph of the amplitude $A$ of the displacement $y$ as a function of $\omega$ (supposing $C$ the same for all $\omega$).

(d) A typical long-period seismometer has a period of about 30 sec and a $Q$ of about 2. As the result of a violent earthquake the earth's surface may oscillate with a period of about 20 min and with an amplitude such that the maximum acceleration is about $10^{-9}$ m/sec$^2$. How small a value of $A$ must be observable if this is to be detected?

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*4–15* The free oscillations of a mechanical system are observed to have a certain angular frequency $\omega_1$. The same system, when driven by a force $F_0 \cos \omega t$ (where $F_0$ = const. and $\omega$ is variable), has a power resonance curve whose angular frequency width, at half-maximum power, is $\omega_1/5$.

(a) At what angular frequency does the maximum power input occur?

(b) What is the $Q$ of the system?

(c) The system consists of a mass $m$ on a spring of spring constant $k$. In terms of $m$ and $k$, what is the value of the constant $b$ in the resistive term $-bv$?

(d) Sketch the amplitude response curve, marking a few characteristic points on the curve.

*4–16* For the electrical system in the figure, find

(a) The resonant frequency, $\omega_0$.

(b) The resonance width, $\gamma$.

(c) The power absorbed at resonance.